THE CHINESE UNIVERSITY OF HONG KONG

DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018

Tutorial Classwork 8

1. Let X be a T_1 connected space. Show that X is either a singleton or an infinite set.

2. Let $p : X \to Y$ be a quotient map. Suppose that Y is connected and $p^{-1}(\{y\})$ is connected for any $y \in Y$. Show that X is also connected.

3. (a) Let A and B be proper subsets of the topological spaces X and Y respectively. Show that if X and Y are connected, then $(X \times Y) \backslash (A \times B)$ is also connected.

   (b) * Hence, show that $\mathbb{R}^n$ is not homeomorphic to $\mathbb{R}$ for any $n > 1$.

   (Hint: $\mathbb{R} \backslash \{0\}$ is disconnected.)